1 Financial mathematics

Example 1.1. Mrs Y will need 150 000 CZK after 5 years to pay for a transfer of the "flat rights". The bank offers a term account with nominal interest rate 4.4% credited quarterly. How much money must Mrs Y save to cover the necessary amount for the transfer?

Example 1.2. Mr Z bought a house for 3 600 000 Kč using mortgage loan of amount 2 500 000 CZK. He will repay 16 000 CZK monthly in arrear over 25 years. What is the nominal annual interest rate which the bank offered to Mr Z?

Example 1.3. Mrs Y makes deposits of 100 at time 0, and \( x \) at time 3. The fund grows at a force of interest \( \delta_t = \frac{t^2}{100}, \ t > 0 \).

Let the amount of interest earned from time 3 to 6 is also equal to \( x \). Calculate \( x \).

Example 1.4. Mr X wants to borrow 150 000$. He would like to repay this loan in 2 years by periodic semiannual payments. Bank offers the nominal interest rate 6.9%. Mr X has 45 000$ on his account where the interest is credited monthly under nominal interest rate 2.5%. He can save 40 000$ every half a year from his salary.

0. Plot cash-flow.

1. Will Mr X have enough money on his account to cover the semiannual loan payments?

2. How much money should he hold at the beginning to cover the loan payments?

Solution: 2. Loan repayments \( R \), savings \( S = 40000 \), nominal interest rate \( j_{(12)} = 0.025 \).

Cash-flow balance equation: future value after 2 years should be zero ...

\[
FV_{2y} = 0 = \left\{ \left( \text{Init. money} \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S \right) \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S \right\} \ldots
\]

\[
\ldots \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S \right\} \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S.
\]

This leads to

\[
\text{Initial money} = \sum_{i=1}^{4} \frac{(R - S)}{\left( 1 + \frac{j_{(12)}}{12} \right)^{6i}}
\]
Example 1.5. Mr X sold his car for 200 000 CZK. He paid this amount to his account where the interest is credited monthly with nominal interest rate 2.8%. He decided to buy a new car 9 months after. There was a necessary advance payment of 50 000 CZK taken from the account. Then, the debt was repaid by payments 9 000 CZK monthly. How long the money on the account can cover these payments?

Solution:

\[ 200000 \left( 1 + \frac{0.028}{12} \right)^9 - 50000 = 9000 \alpha_n^{(12)}. \]

Express \( n \).

Example 1.6. Calculate the net present value for a bond with the nominal value 1000 $, annual coupon rate 6% and term to maturity 3 years. Consider a yield curve with annual spot/forward interest rates 3, 4, 5 %.

Solution:

\[ PV = \frac{60}{1 + 0.03} + \frac{60}{(1 + 0.03)(1 + 0.04)} + \frac{1000 + 60}{(1 + 0.03)(1 + 0.04)(1 + 0.05)}. \]