Homework 3 – Nonlinear programming in portfolio optimization

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Computational Aspects of Optimization
Consider $n$ assets with random rates of return denoted by $R_i$, with $\mathbb{E}|R_i|^3 < \infty$ and define the corresponding covariance matrix $C$ and skewness tensor $S$ elementwise as

$$C_{jk} := \mathbb{E}(R_j - \mathbb{E}R_j)(R_k - \mathbb{E}R_k),$$
$$S_{jkl} := \mathbb{E}(R_j - \mathbb{E}R_j)(R_k - \mathbb{E}R_k)(R_l - \mathbb{E}R_l).$$

Employ the aggregate function approach of multiobjective optimization with aggregation parameter $c > 0$

$$\text{minimize} \quad \sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk}x_jx_k - c \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} S_{jkl}x_jx_kx_l$$

$$\sum_{i=1}^{n} \mathbb{E}[R_i] \cdot x_i \geq r_0,$$

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0.$$
Use the same data as for the CVaR and VaR homework to estimate the mean vector, variance matrix and skewness tensor.

2. Solve the mean–variance ($c = 0$) and the mean–variance–skewness\(^1\) ($c = 0.1$) problems.

3. Solve the problems for different 11 values $r_0 \in \{\min_i R_i, \ldots, \max_i R_i\}$.

4. Plot the optimal values against the corresponding values of $r_0$.

Use both Matlab and GAMS.

\(^1\)The problem is nonconvex in general.