Data Envelopment Analysis in Finance and Energy
– New Approaches to Efficiency and their Numerical Tractability

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Contents

1. Efficiency of investment opportunities
2. Data Envelopment Analysis
3. DEA with diversification
4. Representative portfolio efficiency – an empirical study
Efficiency of investment opportunities

Data Envelopment Analysis

DEA with diversification

Representative portfolio efficiency – an empirical study
Various approaches how to test efficiency of an investment opportunity with a random outcome (profit, loss, etc.):

- von Neumann and Morgenstern (1944): **Utility, expected utility**
- Markowitz (1952): **Mean-variance, mean-risk, mean-deviation**
Our approach combines

- **DEA efficiency** – Murthi et al. (1997), Briec et al. (2004), Lamb and Tee (2012), Branda (2013A, 2013B)

- Extension of **mean-risk efficiency** based on **multiobjective optimization** principles – Markowitz (1952)

- **Risk shaping** – several risk measures included into one model – Rockafellar and Uryasev (2002)
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Charnes, Cooper and Rhodes (1978): a way how to state efficiency of a decision making unit over all other decision making units with the same structure of inputs and outputs.

Let $Z_{1i}, \ldots, Z_{Ki}$ denote the inputs and $Y_{1i}, \ldots, Y_{Ji}$ denote the outputs of the unit $i$ from $n$ considered units. **DEA efficiency** of the unit $0 \in \{1, \ldots, n\}$ is then evaluated using the optimal value of the following program where the weighted inputs are compared with the weighted outputs.

All data are usually assumed to be positive.
Banker, Charnes and Cooper (1984): DEA model with Variable Return to Scale (VRS) or BCC:

\[
\begin{align*}
\text{max} & \quad \frac{\sum_{j=1}^{J} y_{j0} Y_{j0} - y_0}{\sum_{k=1}^{K} w_{k0} Z_{k0}} \\
\text{s.t.} & \quad \frac{\sum_{j=1}^{J} y_{j0} Y_{ji} - y_0}{\sum_{k=1}^{K} w_{k0} Z_{ki}} \leq 1, \quad i = 1, \ldots, n, \\
& \quad w_{k0} \geq 0, \quad k = 1, \ldots, K, \\
& \quad y_{j0} \geq 0, \quad j = 1, \ldots, J, \\
& \quad y_0 \in \mathbb{R}.
\end{align*}
\]
Dual formulation of VRS DEA (more useful):

\[
\begin{align*}
\min_{x_i, \theta} & \quad \theta \\
\text{s.t.} & \quad \sum_{i=1}^{n} x_i Y_{ji} \geq Y_{j0}, \quad j = 1, \ldots, J, \\
& \quad \sum_{i=1}^{n} x_i Z_{ki} \leq \theta \cdot Z_{k0}, \quad k = 1, \ldots, K, \\
& \quad \sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, \quad i = 1, \ldots, n.
\end{align*}
\]
Data envelopment analysis

**DEA** – traditional strong wide area (many applications and theory, Handbooks, papers in highly impacted journals, e.g. Omega, EJOR, JOTA, JORS, EE, JoBF)

- production theory (production possibility set),
- returns to scale (CRS, VRS, NIRS, ...),
- radial/slacks-based/directional distance models,
- fractional/primal/dual formulations,
- multiobjective opt. – strong/weak Pareto efficiency,
- stochastic data – reliability, chance constraints,
- dynamic (network) DEA,
- super-efficiency, cross-efficiency, ...
- the most efficient unit
- ...

M. Branda (Charles University)
Contents

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Inputs and outputs

Efficiency of investment opportunities with random outcomes $R_1, \ldots, R_n$ – not directly used as inputs or outputs, in general

- **Inputs**: characteristics with lower values preferred to higher values,
- **Outputs**: characteristics with higher values preferred to lower values.
We consider $n$ assets and denote $R_i \in \mathcal{L}_2(\Omega)$ the rate of return of $i$-th asset and the sets of investment opportunities:

1. **pairwise efficiency** (investment into one single opportunity):

   $$\chi^P = \{R_i, i = 1, \ldots, n\},$$

2. **full diversification** (diversification across all opportunities):

   $$\chi^{FD} = \left\{ \sum_{i=1}^{n} R_i x_i : \sum_{i=1}^{n} x_i = 1, x_i \geq 0 \right\},$$

3. Short sales and margin requirements, limited diversification, etc.
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3. Short sales and margin requirements, limited diversification, etc.
Rockafellar, Uryasev and Zabarankin (2006A, 2006B): GDM are introduced as an extension of standard deviation but they need not to be symmetric with respect to upside $X - \mathbb{E}[X]$ and downside $\mathbb{E}[X] - X$ of a random variable $X$.

Any functional $\mathcal{D} : \mathcal{L}_2(\Omega) \to [0, \infty]$ is called a general deviation measure if it satisfies

(D1) $\mathcal{D}(X + C) = \mathcal{D}(X)$ for all $X$ and constants $C$,
(D2) $\mathcal{D}(0) = 0$, and $\mathcal{D}(\lambda X) = \lambda \mathcal{D}(X)$ for all $X$ and all $\lambda > 0$,
(D3) $\mathcal{D}(X + Y) \leq \mathcal{D}(X) + \mathcal{D}(Y)$ for all $X$ and $Y$,
(D4) $\mathcal{D}(X) \geq 0$ for all $X$, with $\mathcal{D}(X) > 0$ for nonconstant $X$.

(D2) & (D3) $\Rightarrow$ convexity
General deviation measures

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Deviation measures

- **Standard deviation**
  \[
  D(X) = \sigma(X) = \sqrt{\mathbb{E}\|X - \mathbb{E}[X]\|_2}
  \]

- **Mean absolute deviation**
  \[
  D(X) = \mathbb{E}[|X - \mathbb{E}[X]|].
  \]

- **Mean absolute lower and upper semideviation**
  \[
  D_-(X) = \mathbb{E}[|X - \mathbb{E}[X]|],
  D_+(X) = \mathbb{E}[|X - \mathbb{E}[X]|].
  \]

- **Worst-case deviation**
  \[
  D(X) = \sup_{\omega \in \Omega} |X(\omega) - \mathbb{E}[X]|.
  \]

Mean absolute deviation from \((1 - \alpha)\)-th quantile

CVaR deviation

For any \(\alpha \in (0, 1)\) a finite, continuous, lower range dominated deviation measure

\[
D_{\alpha}(X) = \text{CVaR}_{\alpha}(X - E[X]).
\] (1)

The deviation is also called \textbf{weighted mean absolute deviation from the (1 - \(\alpha\))-th quantile}, see Ogryczak, Ruszczynski (2002), because it can be expressed as

\[
D_{\alpha}(X) = \min_{\xi \in \mathbb{R}} \frac{1}{1 - \alpha} E[\max\{(1 - \alpha)(X - \xi), \alpha(\xi - X)\}]
\] (2)

with the minimum attained at any \((1 - \alpha)\)-th quantile. In relation with CVaR minimization formula, see Pflug (2000), Rockafellar and Uryasev (2000, 2002).
Coherent risk and return measures

CRM: $\mathcal{R} : \mathcal{L}_2(\Omega) \rightarrow (-\infty, \infty]$ that satisfies

(R1) $\mathcal{R}(X + C) = \mathcal{R}(X) - C$ for all $X$ and constants $C$,

(R2) $\mathcal{R}(0) = 0$, and $\mathcal{R}(\lambda X) = \lambda \mathcal{R}(X)$ for all $X$ and all $\lambda > 0$,

(R3) $\mathcal{R}(X + Y) \leq \mathcal{R}(X) + \mathcal{R}(Y)$ for all $X$ and $Y$,

(R4) $\mathcal{R}(X) \leq \mathcal{R}(Y)$ when $X \geq Y$.

Moreover, risk measures multiplied by a negative constant can be used as return functionals, i.e.

$\mathcal{E}(X) = -\mathcal{R}(X)$. 
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Moreover, risk measures multiplied by a negative constant can be used as return functionals, i.e.

\[ \mathcal{E}(X) = -\mathcal{R}(X). \]
Traditional DEA model
Input oriented (VRS)

We assume that $X_0$ is not constant, i.e. $D_k(X_0) > 0$, for all $k = 1, \ldots, K$. Input oriented VRS model can be formulated in the dual form

$$\theta^T(X_0) = \min \theta$$

s.t.

$$\sum_{i=1}^{n} x_i \cdot E_j(R_i) \geq E_j(X_0), \ j = 1, \ldots, J, \quad (3)$$

$$\sum_{i=1}^{n} x_i \cdot D_k(R_i) \leq \theta \cdot D_k(X_0), \ k = 1, \ldots, K,$$

$$\sum_{i=1}^{n} x_i = 1, \ x_i \geq 0, \ i = 1, \ldots, n.$$
The model does not take into account portfolio diversification: For any general deviation measure $D_k$ it holds

$$\sum_{i=1}^{n} x_i \cdot D_k(R_i) \geq D_k\left(\sum_{i=1}^{n} x_i R_i\right)$$

for nonnegative weights with $\sum_{i=1}^{n} x_i = 1$.

Linear transformation of inputs is only an upper bound for the real portfolio deviation.
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for nonnegative weights with $\sum_{i=1}^{n} x_i = 1$.

Linear transformation of inputs is only an upper bound for the real portfolio deviation.
Traditional DEA and diversification frontier
Efficiency of mutual funds or industry representative portfolios:

We assume that the benchmark \( X_0 \in \mathcal{X} \) is not constant, i.e. \( D_k(X_0) > 0 \), for all \( k = 1, \ldots, K \).

**Definition**

We say that \( X_0 \in \mathcal{X} \) is **DEA efficient** with respect to the set \( \mathcal{X} \) if the optimal value of the DEA program is equal to 1. Otherwise, \( X_0 \) is inefficient and the optimal value measures the inefficiency.

Sets of efficient opportunities

\[
\Psi^{I/O} = \{ X \in \mathcal{X} : \theta^{I/O}(X) = 1 \},
\]

where \( \theta^{I/O}(X_0) \) is the optimal value for benchmark \( X_0 \).
For a benchmark $X_0 \in \mathcal{X}$, the input oriented diversification consistent DEA test:

$$\theta^I(X_0) = \min \theta$$

s.t.

$$\mathcal{E}_j(X) \geq \mathcal{E}_j(X_0), \ j = 1, \ldots, J,$$

$$\mathcal{D}_k(X) \leq \theta \cdot \mathcal{D}_k(X_0), \ k = 1, \ldots, K,$$

$X \in \mathcal{X}$. 

\[ (4) \]
Input-output oriented DC DEA models - (in)efficiency is measured also with respect to the outputs (assume $\varepsilon_j(X_0) > 0$):

- optimal values (efficiency scores) and strength can be compared,
- input and input-output oriented models can be compared: I-O tests are stronger in general,

We assume that $\mathcal{E}_j(X_0)$ is positive for at least one $j$. An input-output oriented test where inefficiency is measured with respect to the inputs and outputs separately can be formulated as follows:

$$\theta^{I-O}(X_0) = \min_{\theta, \varphi, X} \frac{\theta}{\varphi}$$

subject to

$$\mathcal{E}_j(X) \geq \varphi \cdot \mathcal{E}_j(X_0), \quad j = 1, \ldots, J,$$

$$\mathcal{D}_k(X) \leq \theta \cdot \mathcal{D}_k(X_0), \quad k = 1, \ldots, K,$$

$$0 \leq \theta \leq 1, \quad \varphi \geq 1,$$

$$X \in \mathcal{X}.$$
Input-output oriented tests

Setting $1/t = \varphi$, results into an input oriented DEA test with nonincreasing return to scale (NIRS):

$$\theta^{I-O}(R_0) = \min_{\tilde{\theta}, \tilde{x}_i} \tilde{\theta}$$

s.t. $\mathcal{E}_j \left( \sum_{i=1}^{n} R_i \tilde{x}_i \right) \geq \mathcal{E}_j(R_0), \ j = 1, \ldots, J,$

$$\mathcal{D}_k \left( \sum_{i=1}^{n} R_i \tilde{x}_i \right) \leq \tilde{\theta} \cdot \mathcal{D}_k(R_0), \ k = 1, \ldots, K,$$

$$\sum_{i=1}^{n} \tilde{x}_i \leq 1, \ \tilde{x}_i \geq 0, \ 1 \geq \tilde{\theta} \geq 0.$$
Properties and relations

Proposition

The considered DEA models are unit invariant.

For arbitrary $k$ and $j$, $\lambda D_k(X) = D_k(\lambda X)$ which implies $E_j(\lambda X) = \lambda E_j(\lambda X)$ for arbitrary $X \in \mathcal{X}$ and $\lambda > 0$. 
Properties and relations

Proposition

Let \( \max\{J, K\} \geq 2 \). Then for a benchmark \( X_0 \in \mathcal{X} \) with \( D_k(X_0) > 0 \) for all \( k \) and \( E_j(X_0) > 0 \) for all \( j \), the following relations hold

\[
\theta^T(X_0) \geq \theta^I(X_0) \geq \theta^{I-O}(X_0).
\]

Then, for the sets of efficient portfolios can be obtained

\[
\Psi^{I-O} \subseteq \Psi^I \subseteq \Psi^T.
\]
Proposition

The optimal solution of the test is efficient with respect to the test.
1. Efficiency of investment opportunities

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4. Representative portfolio efficiency – an empirical study
To compare the efficiency tests, we consider:

- historical US stock market data, monthly excess returns from January 2002 to December 2011 (120 observations) of 48 representative industry stock portfolios that serve as the base assets. The industry portfolios are based on four-digit SIC codes and are from Kenneth French library.

- Portfolio composed from the representative portfolios = interdisciplinary portfolio.
DC DEA test with CVaR deviations
Input oriented

For discretely distributed returns ($r_{is}$, $s = 1, \ldots, S$, $p_s = 1/S$) LP:

$$
\theta^I(R_0) = \min_{\theta, x_i, u_{sk}, \xi_k} \theta \\
\text{s.t.} \quad \sum_{i=1}^{n} \mathbb{E}[R_i] x_i \geq \mathbb{E}[R_0],
$$

$$
\frac{1}{S} \sum_{s=1}^{S} u_{sk} \leq \theta \cdot D_{\alpha_k}(R_0), \quad k = 1, \ldots, K,
$$

$$
u_{sk} \geq \sum_{i=1}^{n} x_i r_{is} - \xi_k, \quad s = 1, \ldots, S, \quad k = 1, \ldots, K,
$$

$$
u_{sk} \geq \frac{\alpha_k}{1 - \alpha_k} \left( \xi_k - \sum_{i=1}^{n} x_i r_{is} \right),
$$

$$
\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, \quad i = 1, \ldots, n.
$$
DC DEA test with CVaR deviations
Input-output oriented

For discretely distributed returns \((r_{is}, s = 1, \ldots, S, p_s = 1/S)\) LP:

\[
\begin{align*}
\theta^{I-O}(R_0) &= \min_{\theta, x_i, u_{sk}, \xi_k} \theta \\
\text{s.t.} & \quad \sum_{i=1}^{n} \mathbb{E}[R_i]x_i \geq \mathbb{E}[R_0], \\
& \quad \frac{1}{S} \sum_{s=1}^{S} u_{sk} \leq \theta \cdot D_{\alpha_k}(R_0), \quad k = 1, \ldots, K, \\
& \quad u_{sk} \geq \sum_{i=1}^{n} x_i r_{is} - \xi_k, \quad s = 1, \ldots, S, \quad k = 1, \ldots, K, \\
& \quad u_{sk} \geq \frac{\alpha_k}{1 - \alpha_k} \left( \xi_k - \sum_{i=1}^{n} x_i r_{is} \right), \\
& \quad \sum_{i=1}^{n} x_i \leq 1, \quad x_i \geq 0, \quad i = 1, \ldots, n.
\end{align*}
\]
### Efficient industry representative portfolios and scores

<table>
<thead>
<tr>
<th></th>
<th>Food</th>
<th>Smoke</th>
<th>Hshld$^1$</th>
<th>Drugs</th>
<th>Mines</th>
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<th>Meals</th>
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$^1$Consumer Goods
# Ranking of the industry representative portfolios

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<th>Food</th>
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