Flow around structures

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Motivation

Our goal is to develop a CFD model for complex geometries – especially for urban areas.

In the first step we developed a 2D laminar model for incompressible isothermal flows.

We also employed some not so common techniques.

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Navier-Stokes equations (nondimensional form)

Momentum equations

$$\frac{\partial \vec{u}}{\partial t} + \operatorname{div}\left(\vec{u} \otimes \vec{u}\right) = -\operatorname{grad} p + \frac{1}{\mathrm{Re}} \operatorname{div} \operatorname{grad} \vec{u} \tag{1}$$

$$\mathrm{Re} = \frac{UL}{\nu} \tag{2}$$

Continuity equation (incompressibility constraint)

$$\operatorname{div} \vec{u} = 0 \tag{3}$$

Fractional step method – Brown, Cortez, Minion (2001)

1. Momentum equation

$$\frac{\vec{u}^* - \vec{u}^n}{\Delta t} + \nabla p^{n-1/2} = -\left[\nabla \cdot (\vec{u}\vec{u})\right]^{n+1/2} + \frac{1}{2\mathrm{Re}} \nabla^2 \left(\vec{u}^* + \vec{u}^n\right) \quad (4)$$

2. Projection onto solenoidal field

$$\Delta t \, \nabla^2 \phi^{n+1} = \nabla \cdot \vec{u}^* \quad (5)$$

$$\vec{u}^{n+1} = \vec{u}^* - \Delta t \, \nabla \phi^{n+1} \quad (6)$$

$$p^{n+1/2} = p^{n-1/2} + \phi^{n+1} - \frac{\Delta t}{2\mathrm{Re}} \nabla^2 \phi^{n+1} \quad (7)$$

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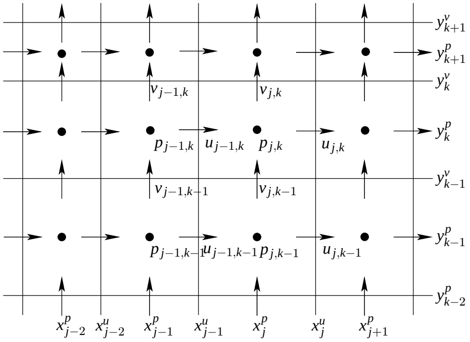
Finite volume method

- Nonuniform Cartesian grid
- MAC staggered placement of variables
- Central differences for gradient and divergence operators



Discretization of advective fluxes

Hyperbolic problem

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0 \tag{8}$$

High-resolution methods

i) At least second order in parts with continuous solution
ii) Do not produce spurious oscillations at discontinuities

Nonlinearity

All high-resolution schemes have to be nonlinear.

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Kurganov-Tadmor scheme

Kurganov, Tadmor (2001)

- Central scheme for hyperbolic conservation laws
- Does not need a Riemann solver and a characteristic decomposition
- Can be applied componentwise
- Semidiscrete - method of lines
- Second order due to a piecewise linear reconstruction
- TVD due to a slope limiter

Kurganov-Tadmor scheme, equations

Semidiscrete form

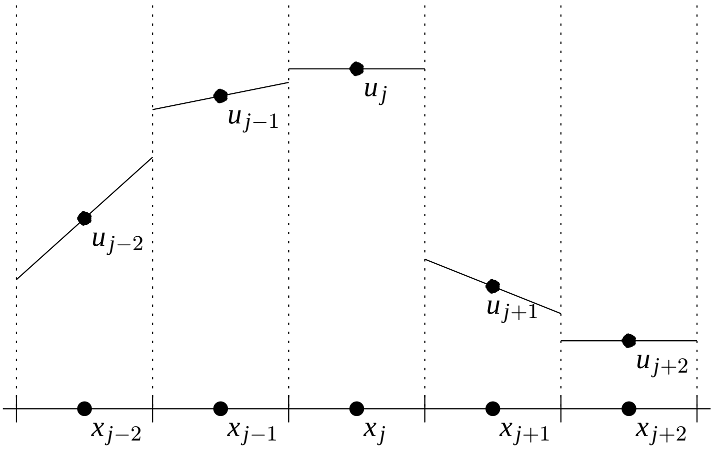
$$\frac{\partial}{\partial t}u_j = -\frac{F_{j+1/2}(u) - F_{j-1/2}(u)}{\Delta x_j} \tag{9}$$

Numerical flux

$$F_{j+1/2} = \frac{f(u^+_{j+1/2}) + f(u^-_{j+1/2})}{2} - \frac{a_{j+1/2}}{2}[u^+_{j+1/2} - u^-_{j+1/2}] \tag{10}$$

- $a_{j+1/2}$ are the local speeds calculated as the greatest eigenvalue of a Jacobian matrix $\frac{\partial f(u)}{\partial u}$
- $u^{\pm}_{j+1/2}$ are the edge vales from the reconstruction

Example of piecewise linear reconstruction



Piecewise linear reconstruction

Slopes

$$u(x) = u_j + s_j(x - x_j) \quad \text{for} \;\; x_{j-1/2} < x < x_{j+1/2} \tag{11}$$

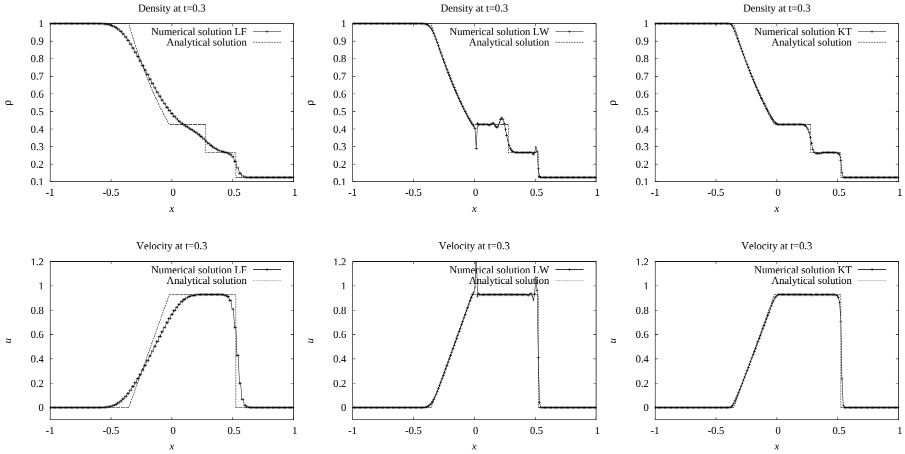
$$s_j^- = \frac{u_j - u_{j-1}}{x_j - x_{j-1}}, \quad s_j^C = \frac{u_{j+1} - u_{j-1}}{x_{j+1} - x_{j-1}} \quad s_j^+ = \frac{u_{j+1} - u_j}{x_{j+1} - x_j} \tag{12}$$

Modified minmod limiter (MC):

$$s_j = \mathrm{minmod}(\theta s_j^-, s_j^C, \theta s_j^+), \;\; \theta \in [1, 2] \tag{13}$$

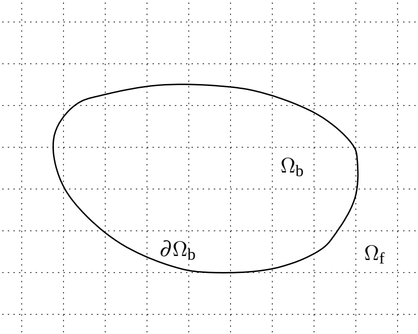
$$\mathrm{minmod}(a, b, c) = \begin{cases} \min(a, b, c) & \text{if } a, b, c \geq 0 \\ \max(a, b, c) & \text{if } a, b, c \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Example of Shock tube problem



Immersed boundary method

- Enables use of Cartesian grid in complex geometries.
- Boundary conditions on solid walls are implemented via additional momentum and mass sources.



Immersed boundary method - Kim, Kim, Choi (2001)

Modified momentum and continuity equations

$$\frac{\vec{u}^* - \vec{u}^n}{\Delta t} + \nabla p^{n-1/2} = -\left[\nabla \cdot (\vec{u}\vec{u})\right]^{n+1/2} + \frac{1}{2\mathrm{Re}} \nabla^2 \left(\vec{u}^* + \vec{u}^n\right) + \vec{f} \tag{15}$$

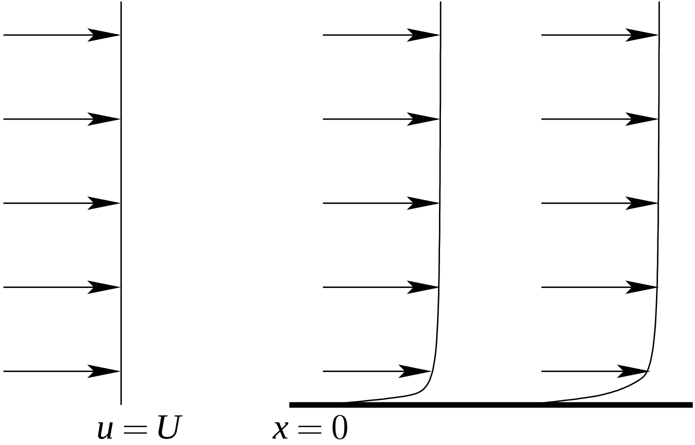
$$\Delta t\, \nabla^2 \phi^{n+1} = \nabla \cdot \vec{u}^* - q \tag{16}$$

Calculation of sources

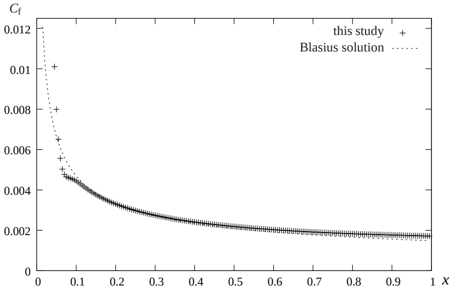
$$\vec{f} = \frac{\vec{U} - \vec{u}^n}{\Delta t} + \left[\nabla \cdot (u\vec{u})\right]^{n+1/2} + \nabla p^{n-1/2} - \frac{1}{\mathrm{Re}} \nabla^2 \vec{u}^n, \tag{17}$$

where $\vec{U}$ is a result of the interpolation from the neighbouring cells to the boundary.

Flow over a flat plate - description

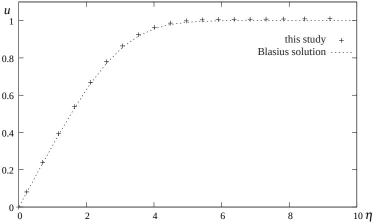


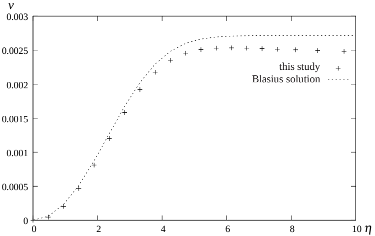
Flow over a flat plate - results



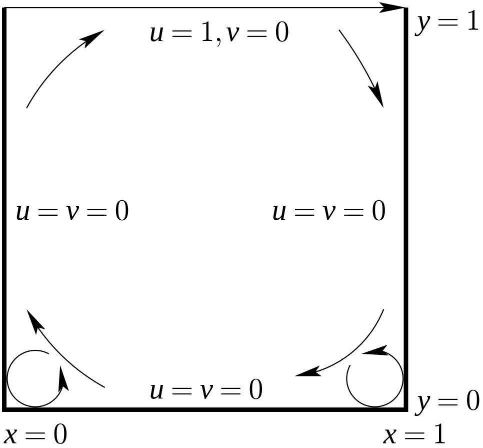
Skin friction coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}, \qquad (18)$$

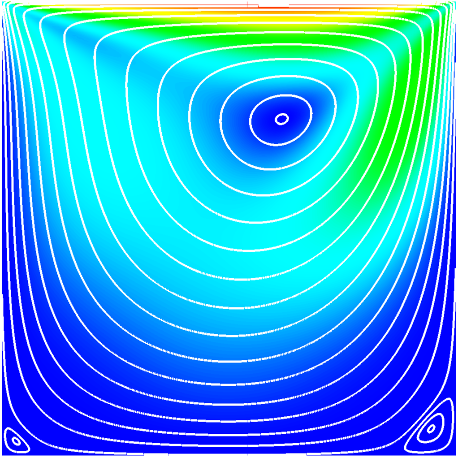


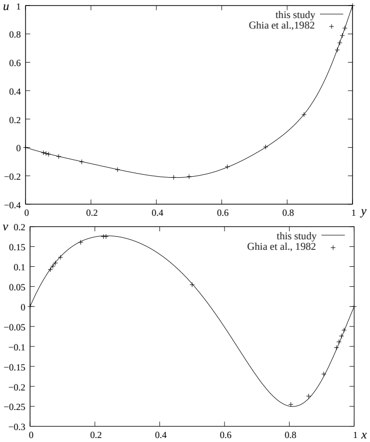


Lid driven cavity - description

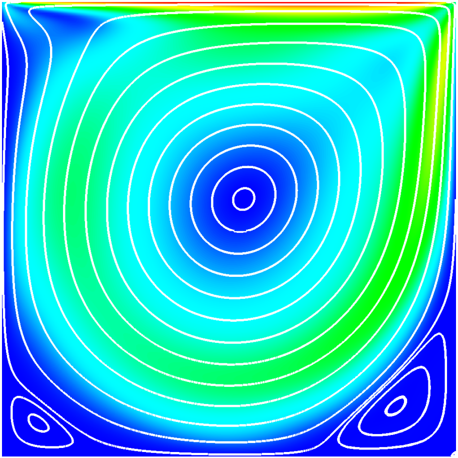


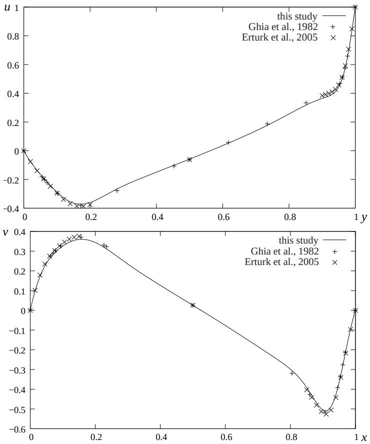
Lid driven cavity $Re = 100$



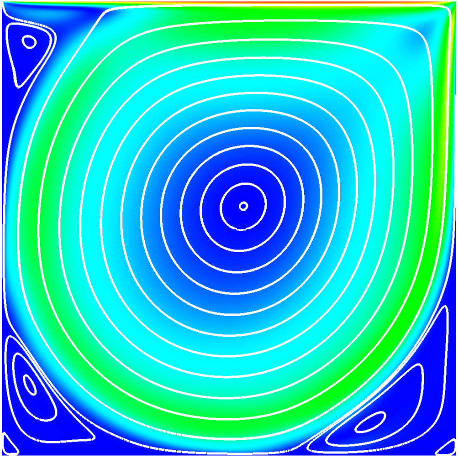


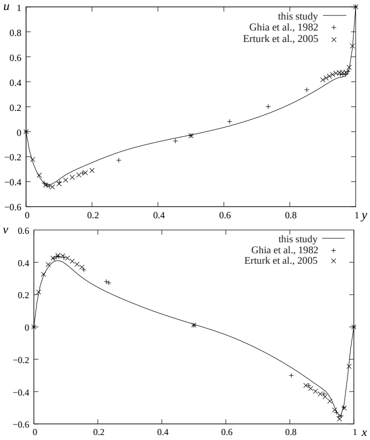
Lid driven cavity Re = 1000



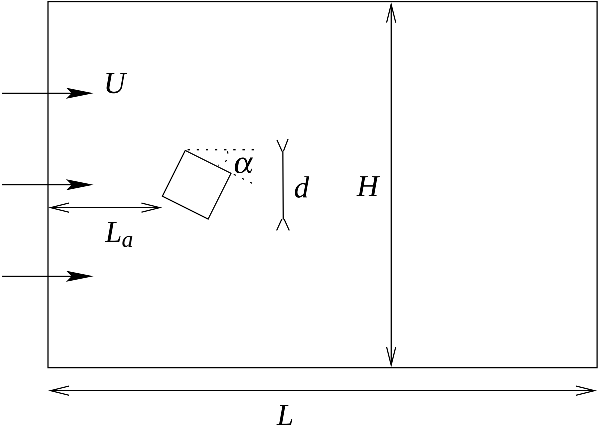


Lid driven cavity Re = 5000





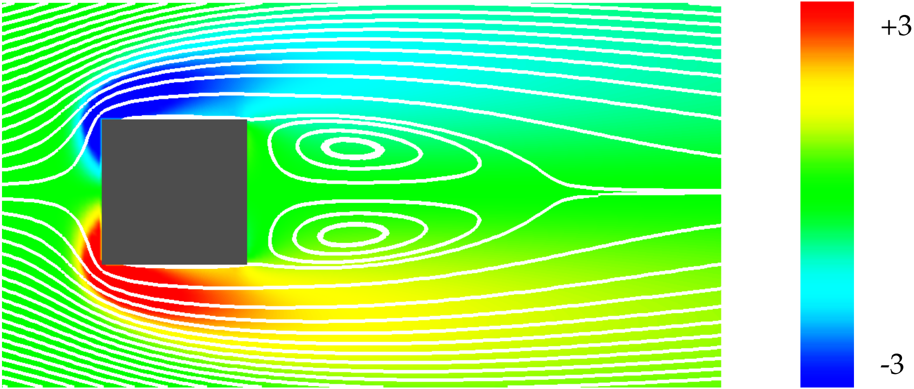
Flow around a sqare cylinder - description



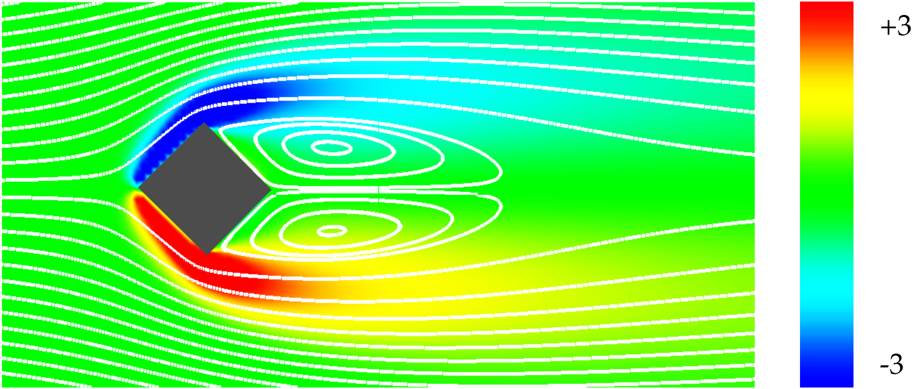
Reynolds number

$$\mathrm{Re} = \frac{Ud}{\nu}$$

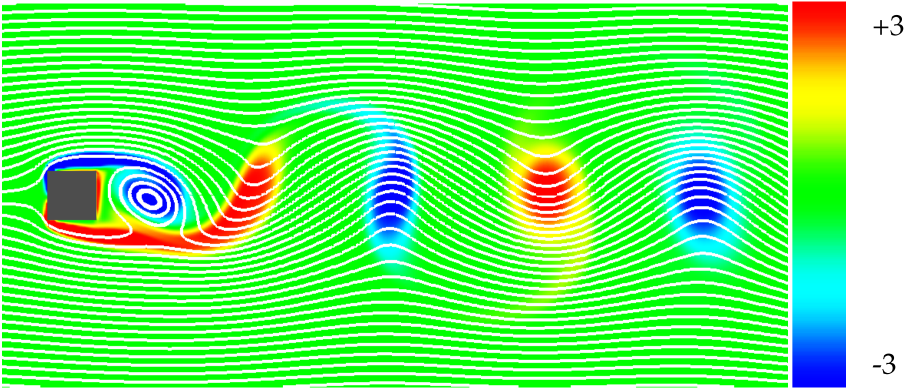
Square cylinder Re = 30, $\alpha = 0°$, vorticity



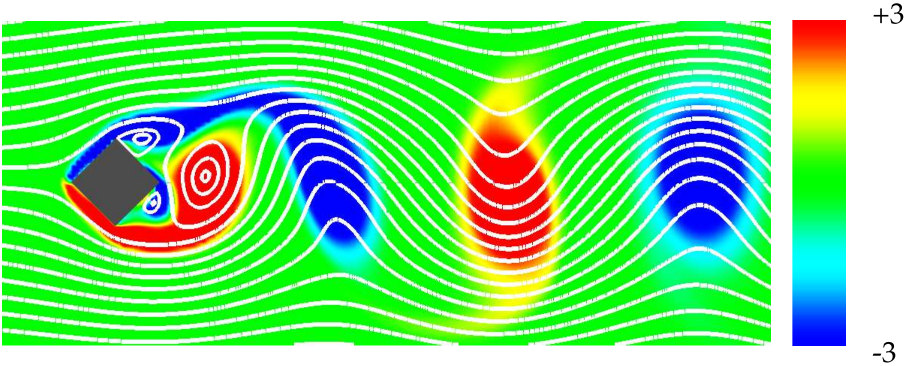
Square cylinder Re $= 30$, $\alpha = 45°$, vorticity



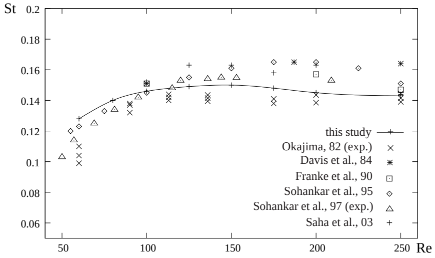
Square cylinder Re $= 200$, $\alpha = 0°$, vorticity

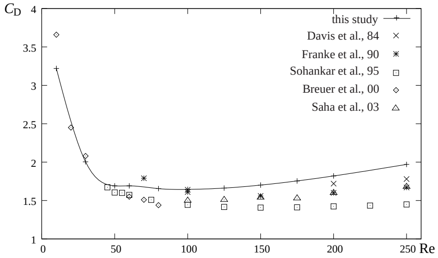


Square cylinder Re $= 200$, $\alpha = 45°$, vorticity



Strouhal number and a drag coefficient for $\alpha = 0°$





Strouhal number

$$\mathrm{St} = \frac{fd}{U}, \qquad (19)$$

where f is a frequency of the vortex shedding

drag coefficient

$$C_\mathrm{D} = \frac{F_\mathrm{D}}{\frac{1}{2}\rho U^2 d}, \qquad (20)$$

where F_D is a drag force

Outlook

Current development

- collocated gid arangement (approximate projection)
- transition to 3D

Planned development

- turbulence modelling – RANS or (M)ILES ?
- stratification
- contaminant dispersion

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Thank you for your attention!

References

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